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Tests of Gravitational Theories Using Space Probes

W. Davidson

University of Otago, Dunedin, New Zealand

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Several high-precision tests of gravitational theories applied to the field of a spherical mass particle (the Sun) are formulated and analyzed. These tests mainly involve the accurate timing by (on-board) atomic clocks of two-way radio signals between two spacecraft (transponders) in various solar orbits. Measures of the gravitational field to at least 1 part in 10^8, or order m/R, where m is the Schwarzschild radius of the Sun and R is the astronomical unit, are aimed at, with an assumed round trip time precision of 10^{-2} μs or 1.5 m distance. Since it is important to distinguish between the radial coordinate parameters occurring in the theory of the tests and their common astronomical measurements based on classical assumptions, two of the tests include the means in principle of determining these coordinate parameters.

1. INTRODUCTION

The testing of general relativity theory by the three classical tests suggested by Einstein (1916) have been fraught with various practical difficulties which have limited the degree of its verification. Only two of these tests were reasonably successful, the deflection of light by the Sun and the rotation of the perihelion of Mercury. Since Einstein's definitive paper of 1916 there have been a considerable number of competing theories put forward, some of which are not easy to distinguish from general relativity by means of the three classical tests. Although general relativity has stood the test of time and competition, and there is today immense confidence in the theory, it is desirable to push the testing to greater limits of accuracy by all modern technological means available.

The first advance on these lines was the satisfactory verification of the gravitational red-shift phenomenon in the field of the Earth by Pound and Rebka (1960), and by Pound and Snider (1964), using the Mössbauer effect

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in the γ-ray emission and resonant absorption by Fe^{57} crystals. This gave agreement with experiment of all gravitational theories incorporating the principle of equivalence, to within 1% of the predicted effect. Radio astronomy, by its development of fine angular resolution by long baseline interferometry, has enabled the testing of the gravitational deflection of electromagnetic waves by the Sun. This involved the monitoring of the position of the radio source 3C 279 when occulted by the Sun (Shramek, 1971; Hill, 1971), and has confirmed the general relativity prediction to within 10%. Using the precision of atomic clocks Shapiro and his co-workers have measured the gravitational "time delay" in the transit of a radio signal passing close to the Sun, by radar reflection off planets such as Mercury at superior conjunction. This has given agreement with general relativity to within 10% (Shapiro et al., 1968; 1971).

In this paper tests will be put forward employing technological means of even greater precision, namely, the measurement by atomic clocks of the transit times of radio signals to and from space probes in various orbits in the solar system. The use of this technique was the basis of a test of general relativity suggested by the author some years ago (Davidson, 1967). The technique has been used in practice in a gravitational time delay test by signals sent from Earth to the orbiting spacecraft Mariner VI and Mariner VII (Anderson et al., 1971). This verified the general relativity prediction to within about 4%. This kind of signaling has been called "active radar" by Thorne and Will (1970), where one of the space probes or transponders retransmits immediately a radio signal received from the other. Unlike the passive radar reflection from planets the active radar technique gives a very precise time of retransmission because of the smallness of the space probe, compared with the wide (and rough) reflecting surface of a planet as well as its uncertain radius. As Thorne and Will have pointed out, the effective time precision by this technique is already 0.1 μs and this can be expected to improve by a factor of 10 within a few years. This would mean a precision in distance between (ideally) stationary transponders of 1.5 m. This is to be compared with the present effective timing accuracy of 10 μs or 1500 m in passive radar with planets. As far as the atomic clocks specifically are concerned, atomic hydrogen maser oscillators have been developed and adapted for space flight. These have a stability of 1 part in 10^{14} over 100 s intervals within periods extending to many hours (NASA Press Release No. 76–106, 1976).

Several of the tests suggested here involve the principle that the space probes move on geodesics of the space-time. In these it may be necessary to deploy a "drag-free" technique already contemplated by NASA. The probe would be contained *in vacuo* by a shell motivated as necessary by small exterior jets so that at all times the probe falls freely in the Sun's field.

2. PRELIMINARY STEPS

We assume that all gravitational theories have dynamics derived from "potentials" g_{ij} in a Riemannian metric so that the Sun's field has the space-time metric described by the expansion, to adequate approximation,

$$ds^2 = \left(1 - \frac{2\alpha m}{r} + \frac{2\beta m^2}{r^2}\right) dt^2 - \left\{\left(1 + \frac{2\gamma m}{r}\right) dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right\} \tag{2.1}$$

where α, β, γ are constants and $m = GM/c^2$ is the gravitational radius of the Sun, G being the Newtonian gravitational constant, M the Sun's mass and c the velocity of light. In general relativity $\alpha = \gamma = 1$, $\beta = 0$. In the Brans–Dicke theory (Brans and Dicke, 1961) we have $\alpha = 1$, $\beta = 1/(\omega + 2)$, $\gamma = (\omega + 1)/(\omega + 2)$ where ω is the dimensionless constant of the theory.

All gravitational theories have to yield the Newtonian equation of motion as first approximation for slow motion in the Sun's field. In terms of the metric this leads to the result

$$g = \frac{\alpha GM}{r^2} \tag{2.2}$$

where g is the acceleration of a test particle. This equation embodies the principle of equivalence and the geodesic principle in the case of general relativity. Since we have no other means of determining the Sun's mass we have effectively to take $\alpha = 1$. The resulting term in the coefficient of dt^2 in the metric may be regarded as verified at least to 1% by the red-shift experiment (for the Earth) already discussed. Henceforth we shall set $\alpha = 1$ in this paper.

The test of the rotation of the perihelion of Mercury brings in the constant β along with γ when the formula is derived from the metric (2.1), using the geodesic hypothesis for planetary orbits. If we ignore the complications raised by a possible quadrupole moment of the Sun (which is now regarded with less favor), then taking $\gamma = 1$ the general relativity prediction of the precession (which then has $\beta = 0$) has been verified to within 2% by the observations of Mercury (Clemence, 1947). However, alternative tests to determine β with greater accuracy are highly desirable, so β and γ will be the unknown constants featuring in the tests of this paper.

If it is possible, as we shall assume henceforth, to plot the "light-time" distance between space vehicles to 1.5 m by active radar, then it is clear that by arranging for probes to circle Earth and other planets such as Mars it should be possible to determine the distance between the center of gravity of the Earth and that of Mars to this accuracy during the entire periods of their orbits. In this way the "light-time" geometry of the solar system as a whole should be known to at least 1 part in 10^{10} accuracy. Consequently we should know to this accuracy when a space vehicle is at rest in the solar system (i.e., maintained in this state by the firing of auxiliary jets as necessary). It is clearly an advantage in formulating the mathematical analysis for tests of gravitational theory if one or more of the vehicles can be regarded as at rest within certain toleration limits. Similarly a space probe may be placed in a circular orbit round the Sun within certain toleration limits as checked from the Earth. These possibilities will be assumed in the subsequent analysis.

3. DETERMINATION OF γ BY A TIME DELAY TEST BETWEEN STATIONARY SPACECRAFT

We imagine two space probes kept at rest on a radius vector from the Sun. The outer one A is fixed at $r = R$ of the metric coordinates (2.1), the inner one P at coordinate $r = \rho$ as near as practicable to the Sun. A sends radio pulses to P which responds with an immediate return pulse. The time of dispatch and receipt of the signal are measured on an atomic clock carried by A, to an accuracy more than adequate for the assumed precision limit 10^{-2} μs. Let a pulse leave A at coordinate time t_1, arrive at P at coordinate time t, and return to A at coordinate time t_2. We assume that the pulse moves on a radial null line of the metric (2.1). It follows that, neglecting terms $O(m^2/r^2)$,

$$\int_\rho^R \left[1 + (1+\gamma)\frac{m}{r}\right] dr = t - t_1 = t_2 - t \tag{3.1}$$

and so

$$t_2 - t_1 = 2\left[R - \rho + (1+\gamma)m \log\frac{R}{\rho}\right] \tag{3.2}$$

The atomic clock at A will register proper time s there and therefore to the required order

$$\begin{aligned} s_2 - s_1 &= \left(1 - \frac{m}{R}\right)(t_2 - t_1) \\ &= 2(R-\rho) + 2m\left[(1+\gamma)\log\frac{R}{\rho} - \frac{R-\rho}{R}\right] \end{aligned} \tag{3.3}$$

Restoring general units this result reads

$$s_2 - s_1 = \frac{2(R-\rho)}{c} + \frac{2\mu}{c^3}\left[(1+\gamma)\log\frac{R}{\rho} - \frac{R-\rho}{R}\right] \tag{3.4}$$

where $\mu = GM$.

The first term is the "Newtonian time" for the two-way trip as if the coordinates ρ, R were Euclidean. What we have to do is to evaluate the coordinates ρ, R so that the Newtonian term can be ascertained to sufficient accuracy to allow determination of the second term. Methods of achieving this in principle will be presented in Sections 4 and 6. The second term in (3.4) is the "gravitational time delay" due to both the gravitational field and the non-Euclidean geometry of space. If one tentatively assigns a value to γ, e.g., $\gamma = 1$ as in general relativity, then knowing ρ and R one may calculate the term. Clearly, even the classical measures of ρ, R will be adequate for this purpose. As an example we set $\gamma = 1$, $\rho = 3R_\odot$, where $R_\odot$ is the classical measure of the Sun's radius and take $R = 1$ AU classically. Following Allen (1973) we take

$$\begin{aligned} c &= 2.9979250(10) \times 10^5 \text{ km s}^{-1} \\ R_\odot &= 6.9599 \times 10^5 \text{ km} \\ 1 \text{ AU} &= 1.495979(1) \times 10^8 \text{ km} \\ m &= GM/c^2 = 1.476 \text{ km} \end{aligned} \tag{3.5}$$

and obtain

$$\begin{aligned} \text{Newtonian time} &= 984.08024 \text{ s} \\ \text{Gravitational time delay} &= 74.42\ \mu\text{s or equivalently } 22.310 \text{ km} \end{aligned} \tag{3.6}$$

The time delay is a significantly measurable quantity under our stated precision limit of 10^{-2} μs (we are ignoring here any imprecision arising from the tolerated departure of the probes from a stationary state). The Newtonian term would need to be evaluated to a greater accuracy than that shown. A precision approaching 10^{-1} μs would be possible if we can determine the coordinates ρ, R to 1 part in 10^{10} (Sections 4 and 6).

4. DETERMINATION OF β AND COORDINATE VALUES r BY USE OF CIRCULAR ORBITS

An ordinary geodesic path in the space-time of metric (2.1) will be assumed. Suppose the orbit of a probe P is in the plane $\phi = \pi/2$. The geodesic equations give

$$r^2 \frac{d\theta}{ds} = h \tag{4.1}$$

$$\left(1 - \frac{2m}{r} + \frac{2\beta m^2}{r^2}\right) \frac{dt}{ds} = k \tag{4.2}$$

h and k being constants. The r equation of a geodesic is

$$\frac{d}{ds}\left[\left(1 + \frac{2\gamma m}{r}\right)\frac{dr}{ds}\right] + \left(\frac{m}{r^2} - \frac{2\beta m^2}{r^3}\right)\left(\frac{dt}{ds}\right)^2 + \frac{\gamma m}{r^2}\left(\frac{dr}{ds}\right)^2 - r\left(\frac{d\theta}{ds}\right)^2 = 0 \tag{4.3}$$

while the metric integral gives

$$\left(1 - \frac{2m}{r} + \frac{2\beta m^2}{r^2}\right)\left(\frac{dt}{ds}\right)^2 - \left(1 + \frac{2\gamma m}{r}\right)\left(\frac{dr}{ds}\right)^2 - r^2\left(\frac{d\theta}{ds}\right)^2 = 1 \quad (4.4)$$

Consider the typical circular geodesic $r = R$. The above equations yield two relations connecting h and k for this case:

$$\frac{m(1 - 2\beta m/R)k^2}{(1 - 2m/R + 2\beta m^2/R^2)^2} = \frac{h^2}{R} \quad (4.5)$$

$$\frac{k^2}{1 - 2m/R + 2\beta m^2/R^2} - \frac{h^2}{R^2} = 1 \quad (4.6)$$

Hence neglecting terms of order m^2/R^2 we obtain

$$h = m^{1/2}R^{1/2}\left[1 + (\tfrac{3}{2} - \beta)\frac{m}{R}\right] \quad (4.7)$$

$$k = 1 - \frac{m}{2R} \quad (4.8)$$

We can now find the proper time for a complete circular orbit as registered by an atomic clock on the spacecraft P. It is

$$[s] = \int_0^{2\pi} \frac{ds}{d\theta}\, d\theta = \frac{R^2}{h}\int_0^{2\pi} d\theta$$

$$= \frac{2\pi R^{3/2}}{m^{1/2}}\left[1 - (\tfrac{3}{2} - \beta)\frac{m}{R}\right]$$

In general units this is

$$[s] = \frac{2\pi R^{3/2}}{\mu^{1/2}}\left[1 - (\tfrac{3}{2} - \beta)\frac{m}{R}\right] \quad (4.9)$$

On the other hand if a probe A was maintained fixed at a point of the same orbit then A's clock would register for P's circuit the time

$$[\bar{t}] = \oint \frac{d\bar{t}}{ds}\, ds = \oint \left(1 - \frac{m}{R}\right)\frac{dt}{ds}\, ds$$

where ds refers to an element of P's world-line. Hence

$$[\bar{t}] = \left(1 - \frac{m}{R}\right)\frac{k}{1 - 2m/R}[s]$$

$$= \frac{2\pi R^{3/2}}{\mu^{1/2}}\left[1 - (1 - \beta)\frac{m}{R}\right] \quad (4.10)$$

In principle, from (4.9) and (4.10) we can derive both β and any value R

of the radial coordinate (γ is absent from both equations). These determinations will be essentially in terms of only one constant m or μ ($= mc^2$). The nature of the experiment would probably require P to be "drag-free" as described in the Introduction, and its maintenance of a circular orbit would have to be carefully monitored from Earth. Because of the high precision of atomic clocks it should then be possible to obtain the coordinate $r = R$ as it occurs in the metric (2.1) to an accuracy of 1 part in 10^{10} in terms of m (~ 1.476 km) as the unit of length. Similarly we obtain β to very high accuracy.

It should be noticed that if we think of R as corresponding in particular to 1 AU then neglect of terms $O(m^2/R^2)$ here and elsewhere in the paper means discarding quantities $O(10^{-16})$. The post-Newtonian terms $O(m/R)$ affect the result for R by the order of a kilometer, and the final result should be accurate to about 10 m (i.e., to 1 part in 10^{10}, approximately) compared with the present uncertainty in the AU of about 10 km (Allen, 1973).

5. DETERMINATION OF γ BY A TIME DELAY TEST BETWEEN ORBITING SPACE PROBES

Let two transponders A and P be placed in the same circular orbit about the Sun, of coordinate radius $r = R$. We shall again think of R as corresponding to 1 AU and the orbit as in the plane of the ecliptic. It will be convenient to have the spacecraft displaced in orbital phase by a little less than 6 months, a precisely known amount to the Earth observer whom we think of as monitoring the spacecraft from the Earth's position approximately midway between in orbital phase. Pulsed signals are sent from A to P so as to pass the Sun at a nearest coordinate distance $r = \rho$. The time of dispatch from A and the arrival at P is registered by their respective on-board atomic clocks. As a consistency check P would immediately retransmit received signals back to A. During the time of the two-way trip the spacecraft would move through only 1.4 arcm of their orbit. Hence for the theoretical calculation it will be sufficiently accurate to take the total time lapse as four times the half-way distance time between A and the point $r = \rho$.

Attention is restricted to theories where the signals move on null geodesics of the space-time (2.1), retaining terms up to $O(m/r)$ only. For the equations in the plane $\phi = \pi/2$ we then get

$$r^2 \frac{d\theta}{d\sigma} = h' \tag{5.1}$$

$$\left(1 - \frac{2m}{r}\right) \frac{dt}{d\sigma} = k' \tag{5.2}$$

where h', k' are constants and σ is an affine parameter along the null geodesic.

We also have the integral from the metric

$$\left(1 - \frac{2m}{r}\right)\left(\frac{dt}{d\sigma}\right)^2 - \left(1 + \frac{2\gamma m}{r}\right)\left(\frac{dr}{d\sigma}\right)^2 - r^2\left(\frac{d\theta}{d\sigma}\right)^2 = 0 \tag{5.3}$$

leading to

$$\begin{aligned}\left(\frac{dr}{dt}\right)^2 &= \frac{1 - 2m/r - (h'^2/k'^2)(1 - 2m/r)^2 r^{-2}}{1 + 2\gamma m/r} \\ &= 0, \qquad r = \rho\end{aligned} \tag{5.4}$$

Hence by elimination of h'^2/k'^2 we obtain to order $O(m/r)$

$$\frac{dr}{dt} = \frac{(r^2 - \rho^2)^{1/2}}{r}\left[1 - \frac{m}{r}\left(1 + \gamma + \frac{\rho}{r + \rho}\right)\right]$$

and so calculate to the required order

$$\begin{aligned}[t]_{r=\rho}^{r=R} &= \int_\rho^R \frac{r}{(r^2 - \rho^2)^{1/2}}\left[1 + \frac{m}{r}\left(1 + \gamma + \frac{\rho}{r + \rho}\right)\right] dr \\ &= (R^2 - \rho^2)^{1/2} + m\left[(1 + \gamma)\cosh^{-1}\left(\frac{R}{\rho}\right) + \left(\frac{R - \rho}{R + \rho}\right)^{1/2}\right]\end{aligned} \tag{5.5}$$

This coordinate interval is related to the required atomic time as registered on A's clock by

$$[s] = [t]\left(\frac{ds}{dt}\right)_A$$

and by the result of Section 4 for a probe moving on a circular orbit we have

$$\frac{dt}{ds} = \frac{k}{1 - 2m/R} \sim 1 + \tfrac{3}{2}\frac{m}{R}$$

by (4.8). Therefore in general units we find for the two-way trip

$$\begin{aligned}[s] = \frac{4}{c}\Bigg\{&(R^2 - \rho^2)^{1/2} \\ &+ m\left[(1 + \gamma)\cosh^{-1}\left(\frac{R}{\rho}\right) + \left(\frac{R - \rho}{R + \rho}\right)^{1/2} - \frac{3}{2}\frac{(R^2 - \rho^2)^{1/2}}{R}\right]\Bigg\}\end{aligned} \tag{5.6}$$

The first term in (5.6) represents the "Newtonian time" as if the coordinates ρ, R were Euclidean and the trajectory a straight line. The remaining terms constitute the "gravitational time delay." This result contains extra terms compared with the formula used in Shapiro et al. (1968, 1971) for their passive radar between the Earth and Mercury. This is because of the very much higher accuracy which can be expected using active radar by spacecraft, as discussed in Section 1. The first term of the delay time is the dominant one but the others are of order $m/c \sim 5\,\mu$s, which is easily measurable by the technique. If we set $\rho = 3R_\odot$, $R = 1$ AU we calculate from classical measurements the round-trip values:

$$\text{Newtonian time} = 1995.82481 \text{ s} \tag{5.7}$$

and (for the case $\gamma = 1$)

$$\text{gravitational time delay} = 185.44\,\mu\text{s} \tag{5.8}$$

Determination of ρ, R to high precision on the lines discussed in Sections 4 and 6 would give the Newtonian time to the high accuracy required for a precise evaluation of γ.

6. DETERMINATION OF β,γ AND EVALUATION OF THE RADIAL COORDINATE $r = R$ BY MEANS OF A PROBE FREELY FALLING TOWARDS THE SUN

Suppose a probe P is moving in a radial path in free fall towards the Sun. Because of the nongravitational forces of solar radiation and solar wind it may be necessary to make the probe "drag-free." By keeping a base spacecraft A at rest at $r = R$ on the same radius vector P's motion can be monitored by signals sent from A and retransmitted from P back to A.

It will be assumed that the free fall is that of an ordinary geodesic in the field of metric (2.1). The geodesic equation in t yields

$$\frac{dt}{ds} = \frac{k}{1 - 2m/r + 2\beta m^2/r^2} \tag{6.1}$$

The constant k may be interpreted in terms of the constant of motion of the probe. Suppose the square of the speed "at infinity" is V^2 (which will be positive or negative according as the motion is hyperbolic or elliptic). Then

$$\frac{1}{k^2} = 1 - V^2 \tag{6.2}$$

The integral provided by the metric itself is

$$\left(\frac{ds}{dt}\right)^2 = 1 - \frac{2m}{r} + \frac{2\beta m^2}{r^2} - \left(1 + \frac{2\gamma m}{r}\right)\left(\frac{dr}{dt}\right)^2 \tag{6.3}$$

Hence, neglecting terms $O(m^2/r^2)$ and restoring general units to clarify the order of the terms, we find from (6.1) and (6.3)

$$\left(\frac{dr}{dt}\right)^2 = V^2 + \frac{2\mu}{r}\left[1 - (\gamma + 2)\frac{V^2}{c^2} - \frac{m}{r}(\beta + 2\gamma + 2)\right] \tag{6.4}$$

Now we can define *operationally* the coordinates $\bar{r}$, $\bar{t}$ by (cf. Section 3)

$$\bar{r} = \frac{s_2 - s_1}{2} = \left(1 - \frac{2m}{R} + \frac{2\beta m^2}{R^2}\right)^{1/2}\left(\frac{t_2 - t_1}{2}\right) \tag{6.5}$$

$$\bar{t} = \frac{s_1 + s_2}{2} = \left(1 - \frac{2m}{R} + \frac{2\beta m^2}{R^2}\right)^{1/2}\left(\frac{t_1 + t_2}{2}\right) \tag{6.6}$$

Retention now of terms up to $O(m/r)$ means we can use the results (3.1), (3.2) on replacing ρ by r, so that

$$\frac{d\bar{r}}{d\bar{t}} = -\frac{dr}{dt}\left[1 + (\gamma + 1)\frac{m}{r}\right] \tag{6.7}$$

Hence by (6.4), (6.7) to the same order we have

$$\left(\frac{d\bar{r}}{d\bar{t}}\right)^2 = V^2 + \frac{2\mu}{r}\left(1 - \frac{V^2}{c^2} - \frac{\beta m}{r}\right) \tag{6.8}$$

where by (6.5) and (3.3) r is given in terms of $\bar{r}$ by

$$\bar{r} = R - r + m\left[(1 + \gamma)\log\frac{R}{r} - \frac{R - r}{R}\right] \tag{6.9}$$

Hence to a consistent order we find that

$$\left(\frac{d\bar{r}}{d\bar{t}}\right)^2 = V^2 + \frac{2\mu}{R - \bar{r}} \times \left\{1 - \frac{V^2}{c^2} - \frac{\mu}{c^2(R - \bar{r})}\left[\beta + (\gamma + 1)\log\left(\frac{R}{R - \bar{r}}\right) - \frac{\bar{r}}{R}\right]\right\} \tag{6.10}$$

an entirely operational equation with parameters R, γ, and β. This result was obtained for the special case of general relativity in an earlier paper (Davidson, 1967).

It is interesting to compare (6.10) with the corresponding Newtonian operational equation, based on Newtonian gravitation, classical kinematics, and a finite velocity of light c:

$$\left(\frac{d\bar{r}}{d\bar{t}}\right)^2 = V^2 + \frac{2\mu}{R - \bar{r}} \tag{6.11}$$

In principle, sufficient data in the form of number pairs ($\bar{r}$, $d\bar{r}/d\bar{t}$) would identify the required parameters. A particularly interesting case arises when a small probe P is launched with hyperbolic velocity, in the range $V^2 \gg 10^2(\mu/R)$. Then (6.10) can be written to good approximation

$$\left(\frac{d\bar{r}}{d\bar{t}}\right)^2 = V^2 + \frac{2\mu}{R - \bar{r}}\left(1 - \frac{V^2}{c^2}\right) \tag{6.12}$$

and the derived acceleration equation in operational coordinates is

$$\frac{d^2\bar{r}}{d\bar{t}^2} = \frac{\mu}{(R - \bar{r})^2}\left(1 - \frac{V^2}{c^2}\right) \tag{6.13}$$

Either equation, independent of β and γ, could be tested to high accuracy by accumulation of data pairs ($\bar{r}$, $d\bar{r}/d\bar{t}$), assuming R was known to the requisite precision. For these equations, if R was of order 1 AU, only moderate precision would be necessary—about 1 part in 10^7. Then (6.12) could be used to obtain R to much higher precision assuming its validity, independent of β and γ, for strongly hyperbolic motions.

7. CONCLUDING REMARKS

The tests discussed in this paper have been put forward as specimen tests of gravitational theory of the type that might be deployed to utilize the far greater precision in time-distance measurements now available in the solar system. This involves radio pulse signaling between probes carrying atomic clocks. Because of the great labor involved in working out the theoretical consequences of general relativity theory, and because of its many controversial features such as black holes currently being discussed, and not least because of its near indispensability in tackling the cosmological problem, it is of paramount importance to push the verification of the theory in all observational aspects to the highest possible precision.

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